Numerical Steady-State Analysis of Nonlinear Microwave Circuits with Periodic Excitation

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Abstract —**A new method for determining the steady-state response of nonlinear microwave circuits with periodic excitation is proposed. The method minimizes time-domain calculations by introducing a criterion for selecting the variables to be considered as unknowns and for solving the resulting nonlinear system by a new and efficient algorithm. It has exhibited the capability for handling a large number of harmonics and nonlinearities. To illustrate the generality and usefulness of the method, a pumped diode and a MESFET frequency doubler are analyzed.**

I. Introduction

The OPTIMUM design of microwave circuits containing nonlinear solid-state devices requires an accurate technique for predicting their nonlinear performance. The most common techniques are based on the analysis of a circuit-type model which simulates the nonlinear behavior of the device. Much work has been done on microwave solid-state device modeling and it is possible to find appropriate models for practically any device. However, the high computational cost of the numerical methods used to analyze the interaction with the external circuit is the major drawback of these techniques.

Nonlinear microwave circuits have two important features: 1) the device-external circuit model usually includes many linear elements; and 2) in most cases the excitation is periodic and only the steady-state response is required. The harmonic balance method is preferable to time-domain techniques because it avoids the numerical integration of the circuit dynamic equations, but it has a serious disadvantage in the large number of unknown variables.

In order to reduce the number of unknown variables, several authors have proposed separating the nonlinear network into linear and nonlinear subnetworks, and considering as unknowns the voltages/currents [1], [2] or the power waves [3] at all the terminals. However, no general rules for optimum circuit partitioning have been given. After partitioning, frequency-domain and time-domain equations are written for the linear and nonlinear subnetworks, respectively. The response of the network is then described by a system of nonlinear equations whose unknowns are the harmonic components of the electrical magnitudes at the terminals.

Several numerical techniques have been employed to solve this nonlinear system. Nakhla and Vlach [1] proposed using a gradient technique for minimizing a certain error function. Filicori *et al.*, [3] used the Newton–Raphson method and avoided convergence problems by increasing step-by-step the nonlinearities of the system. Both methods consume excesive computer time because they must calculate derivatives and, as a result, they become impracticable when many harmonics and/or nonlinearities are considered. One interesting technique has been reported by Hicks and Khan [2], which has exhibited good convergence characteristics when a large number of harmonics are considered and only does one calculation of the functions per iteration.

In this paper, an analysis method is described which avoids the partitioning problem by introducing a criterion for selecting the variables to be considered as unknowns and solving the resulting nonlinear system by a new and efficient algorithm. This method reduces time-domain analysis to the computation of currents and/or voltages at the nonlinear elements from the variables they depend on, and consequently, takes full advantage of the linearities of the network. The waveguide diode mixer analyzed by Kerr [4] is used to compare the iteration algorithm herein proposed with the one by Hicks and Khan. As a demonstration of the capability and usefulness of the method, a general nonlinear MESFET problem including large-signal amplifiers, frequency converters, and harmonics generators is studied. One application, a MESFET frequency doubler, is completely analyzed.

II. Method

Consider the situation represented in Fig. 1, where an M-port arbitrary network, which contains both linear and nonlinear elements, is excited by M periodic sources (P-voltage generators and Q-current generators, hence $M = P + Q$) all with the same period. It is assumed that a steady-state solution exists and the objective is to find it.

Every nonlinear element of the network can be considered either as a voltage generator or as a current generator,

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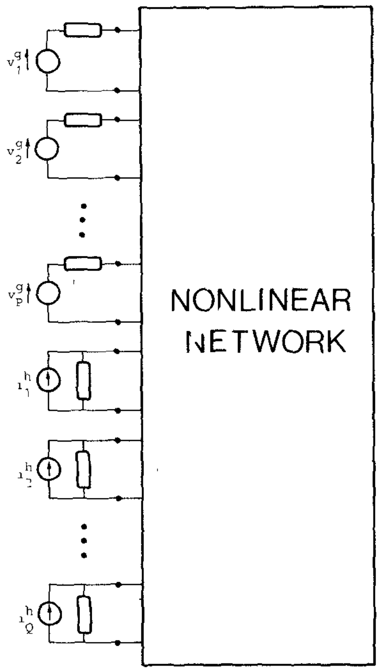

Fig. 1. The general nonlinear problem.

controlled by other voltages and/or currents of the circuit. Let $T+U$ be the number of nonlinear elements (T-voltage generator-type elements and U-current generator-type elements) and let $v_1^x(t), v_2^x(t),\cdots,v_R^x(t), i_1^y(t), i_2^y(t),\cdots,i_S^y(t)$ be the voltages and currents controlling all the nonlinear elements. The aim of the method is to consider these voltages and currents as the unknown variables. Note that, by this way, time-domain analysis is reduced to the computation of the response (voltage or current) of every nonlinear element from the magnitudes it depends on and that the nonlinear problem is solved if these magnitudes are determined.

The circuit in Fig. 1 can be rearranged in the way indicated in Fig. 2 where a $(M+R+S+T+U)$-port linear network, which includes all the linear elements of the primitive circuit, has M ports excited by independent sources, R ports open-circuited, S ports short-circuited, and each of the other $T+U$ ports loaded by one nonlinear element. The voltages and currents at the open-circuited and short-circuited ports, respectively, are the variables controlling all the nonlinear elements. If these magnitudes are known, voltages and currents at the nonlinear elements can be calculated and, after that, any electrical magnitude of the circuit can be obtained by linear transformations.

If the network is in the steady-state with periodic response of period T_0, there will only be nf_0 (n-integer) frequency components in the circuit and every magnitude can be expressed by Fourier series

$$x(t)=\sum_{n=-\infty}^{\infty} X_n \exp(jn\omega_0 t) \tag{1}$$

with $\omega_0 = 2\pi/T_0 = 2\pi f_0$.

According to Fig. 2, it is possible to write for every frequency of interest

$$\begin{bmatrix} V_{1,n}^x \\ V_{2,n}^x \\ \vdots \\ V_{R,n}^x \\ I_{1,n}^y \\ I_{2,n}^y \\ \vdots \\ I_{S,n}^y \end{bmatrix} = [A_n]\cdot \begin{bmatrix} V_{1,n}^v \\ V_{2,n}^v \\ \vdots \\ V_{T,n}^v \\ I_{1,n}^i \\ I_{2,n}^i \\ \vdots \\ I_{U,n}^i \\ V_{1,n}^g \\ V_{2,n}^g \\ \vdots \\ V_{P,n}^g \\ I_{1,n}^h \\ I_{2,n}^h \\ \vdots \\ I_{Q,n}^h \end{bmatrix} \tag{2}$$

where $V_{j,n}^x$, $I_{j,n}^y$, $V_{j,n}^v$, $I_{j,n}^i$, $V_{j,n}^g$, and $I_{j,n}^h$ are the Fourier coefficients of the functions $v_j^x(t)$, $i_j^y(t)$, $v_j^v(t)$, $i_j^i(t)$, $v_j^g(t)$, and $i_j^h(t)$, respectively, and $[A_n]$ is a matrix of $(R+S)x(T+U+M)$ elements obtained by linear analysis of the network at the frequency nf_0.

Since $V_{1,n}^v, V_{2,n}^v,\cdots,V_{T,n}^v, I_{1,n}^i, I_{2,n}^i,\cdots,I_{U,n}^i$ are nonlinear functions of $v_1^x(t), v_2^x(t),\cdots,v_R^x(t), i_1^y(t), i_2^y(t),\cdots,i_S^y(t)$, the relation (2) is equivalent to an infinite system of nonlinear equations of the form

$$X_{i,n} = F_{i,n}(\overline{X}_1, \overline{X}_2,\cdots,\overline{X}_{R+S}) \tag{3}$$

where

i $1,2,\cdots,R+S$
n $0,1,2,\cdots$
$\overline{X}_i$ $(X_{i,1}, X_{i,2},\cdots)$.

If only N harmonics are considered the problem is reduced to solving a system of $(N+1)(R+S)$ nonlinear equations. Its solution can be numerically found using the iteration technique defined by the expression

$$(X_{i,n})_{k+1} = (F_{i,n})_k + \frac{[(F_{i,n})_k-(F_{i,n})_{k-1}]\cdot[(X_{i,n})_k-(F_{i,n})_k]}{[(F_{i,n})_k-(X_{i,n})_k]-[(F_{i,n})_{k-1}-(X_{i,n})_{k-1}]}. \tag{4}$$

Note that the proposed iteration technique is a direct iteration "corrected" to take into account the behavior of the functions in the last two iterations and that it is only necessary to compute the values of the functions $F_{i,n}$ at each step.

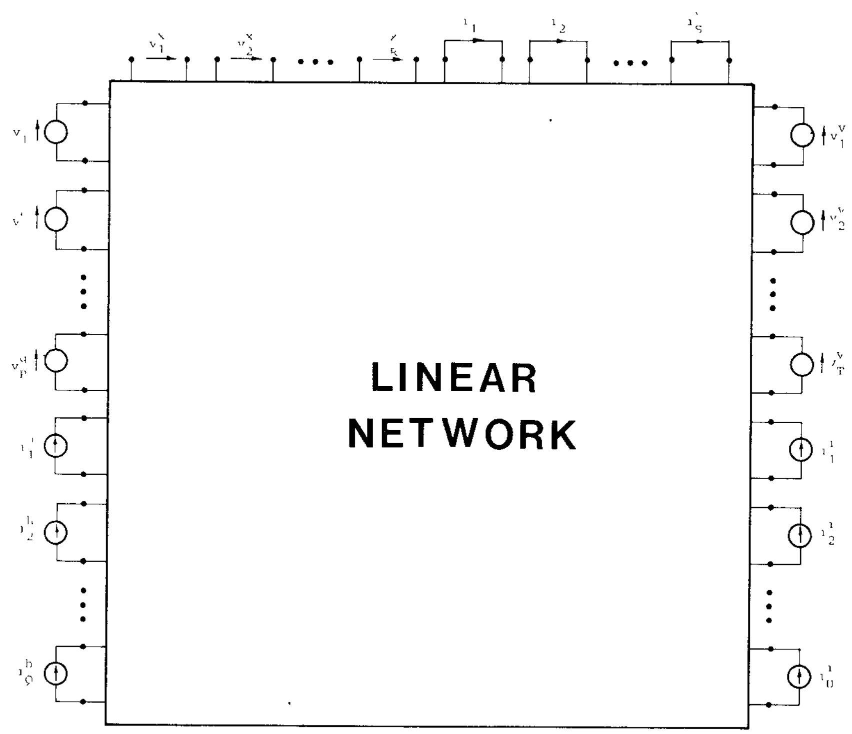

Fig. 2. The general nonlinear problem (rearranged).

The iteration formula (4) fails if the denominator of the "correction factor" is equal to zero. In this case a direct iteration is used, i.e.,

$$(X_{i,n})_{k+1} = (F_{i,n})_k. \tag{5}$$

For the two first iterations the formula (4) is not defined and, consequently, it is necessary to assign appropriate initial values. A choice which has given excellent results is the following.

1) First Iteration: Assign to $X_{i,n}$ the values obtained when all voltage-generator-type elements are short-circuited and all the current-generator-type elements are open-circuited, i.e., setting $v_1^v(t) = v_2^v(t) = \cdots = v_T^v(t) = i_1^i(t) = i_2^i(t) = \cdots = i_U^i(t) = 0$.

2) Second Iteration: Use a direct iteration, i.e., $(X_{i,n})_2 = (F_{i,n})_1$.

Finally, note that in many cases the nonlinear element characteristics are such that the element may be considered as a voltage-generator type or as a current generator type. When a truncation is performed, different solutions are obtained depending on the choice. Since truncation of voltages implies short-circuiting (open-circuiting for currents) the harmonics not considered, the choice of the generator type is suggested by the loading conditions established by the circuit at these harmonics.

III. Application to the Nonlinear Analysis of a Pumped Diode

The waveguide diode mixer analyzed by Kerr [4] was used by Hicks and Khan [5] to show the speed advantage their method has over Kerr's approach. The same example has been selected to compare the iteration algorithm described in this paper with the one proposed by Hicks and Khan.

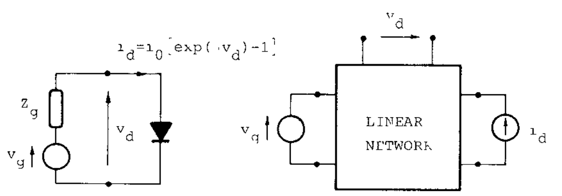

Fig. 3. (a) Equivalent circuit of Kerr's waveguide diode mixer. (b) The same circuit but rearranged. Parameter values are: $i_0 = 5\ nA$, $\alpha = 40\ V^{-1}$.

The equivalent circuit of Kerr's waveguide mixer is represented in Fig. 3(a). The values of $Z_g(f)$ corresponding to the 16 harmonics considered in the analysis can be found in [4]. Since this impedance approaches short-circuit conditions with increasing frequency, the voltage $v_d(t)$ must be selected as unknown variable. Fig. 3(b) shows the circuit after the rearrangement described in Section II.

It is evident that the harmonic components of the unknown are given by

$$[V_{d,n}] = \begin{bmatrix} -Z_{g,n} & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{d,n} \\ V_{g,n} \end{bmatrix} \tag{6}$$

where $V_{d,n}$, $V_{g,n}$, and $I_{d,n}$ are the Fourier coefficients corresponding to $v_d(t)$, $v_g(t)$, and $i_d(t)$, respectively, and $Z_{g,n}$ denotes the value of $Z_g(f)$ at nf_0.

For solving the nonlinear system Hicks and Khan have proposed the iteration algorithm defined by

$$(X_n)_{k+1} = p_n(F_n)_k + (1-p_n)(X_n)_k \tag{7}$$

where p_n is determined by convergence considerations [2]. These authors point out that no significant improvement is

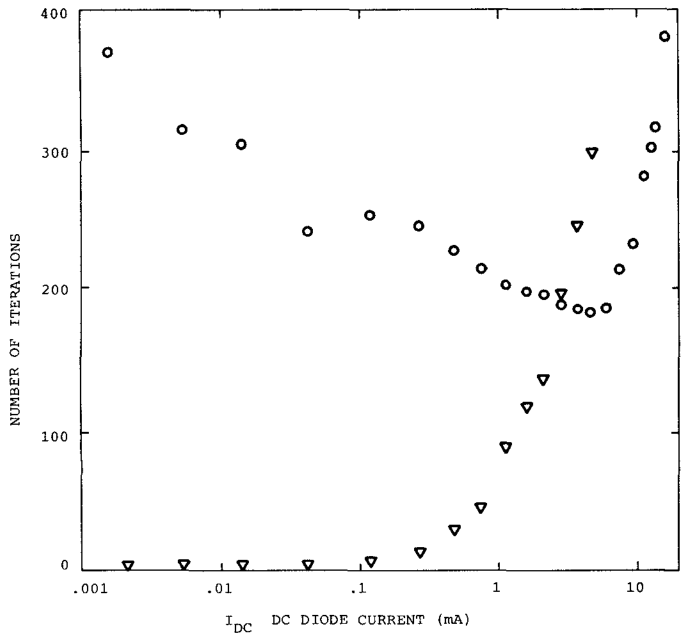

Fig. 4. Number of iterations required for solving Kerr's waveguide diode mixer (16 harmonics) versus dc diode current (O—Hicks–Khan's algorithm with $p = .025$; ▽—proposed algorithm).

achieved by using variable and complex p_n over constant and real p_n. Thus they proposed to use a real value of p valid for all n. Probably after some trials, they set $p = 0.025$ [5] for the problem under consideration, but other situations could require a different optimum value for this parameter.

On the other hand, it is easy to prove that expression (4) is equivalent to (7) with

$$p_n = \frac{1}{1 - \dfrac{(F_n)_k - (F_n)_{k-1}}{(X_n)_k - (X_n)_{k-1}}}. \tag{8}$$

Thus in the proposed algorithm p_n is complex and takes different values for each iteration.

Fig. 4 shows the number of iterations required to reach the solution by both methods for several injection levels. No more than 400 iterations were allowed. Convergence is achieved when the voltage reflection coefficient between the boundary condition impedance $((V_{g,n} - Z_{g,n} I_{d,n})/I_{d,n})$ imposed to the device by the external circuit and the device impedance $(V_{d,n}/I_{d,n})$ is (in magnitude) less than 0.01 (-40 dB) at every harmonic.

Note that the proposed algorithm is more efficient than Hicks–Khan's with fixed p up to $I_{dc} \simeq 3$ mA. The advantages of the latter technique above this value of diode current are uncompensated by the high number of iterations it needs at low injection levels. The Hicks–Khan algorithm can be made faster by choosing p appropriately for each injection level. However, the proposed algorithm does not require any parameter to be chosen and represents an excellent compromise between good convergence characteristics at both low and high injection levels.

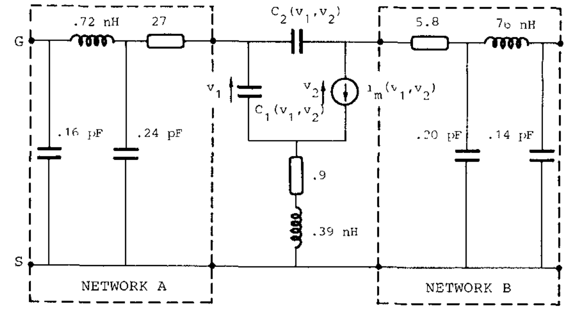

Fig. 5. Nonlinear MESFET model.

IV. Application to the Nonlinear Analysis of Microwave MESFET's

Due to the great interest of MESFET large-signal circuits, a general nonlinear MESFET problem, which includes harmonics generators, frequency converters, and large-signal amplifiers, has been used to demonstrate the capability and usefulness of the method.

A. Nonlinear MESFET Model

The model employed to simulate the nonlinear behavior of the device (NE24406 MESFET by N.E.C.) is shown in Fig. 5. The nonlinear time-invariant elements are the capacitances $C_1(v_1, v_2)$ and $C_2(v_1, v_2)$, and the current generator $i_m(v_1, v_2)$; these elements depend on the voltages v_1 and v_2 while the other ones are linear (they have constant values). Following Rauscher and Willing [6], the instantaneous current through the nonlinear capacitances are obtained by

$$i_C(t) = C[v_1(t), v_2(t)] \frac{dv_C(t)}{dt}. \tag{9}$$

The elements of this nonlinear model have been determined from the static $I_{DC} - V_{DS}$ characteristics and the measured small-signal S-parameters (2–12 GHz) at different bias conditions [7]. A two-dimensional interpolation [8] enables the values of the functions $C_1(v_1, v_2)$, $C_2(v_1, v_2)$, and $i_m(v_1, v_2)$ to be calculated at every point. Extrapolations have been used outside the characterization ranges. This nonlinear MESFET model has shown to be valid at least up to 12 GHz (maximum check frequency).

B. Nonlinear Analysis

The general structure of a wide family of large-signal MESFET circuits, which includes harmonics generators, frequency converters, and large-signal amplifiers, is represented in Fig. 6(a). In this figure V_{LO_1} and V_{LO_2} are periodic sources, V_{B_1} and V_{B_2} are dc bias, and the networks I and II are arbitrary linear networks.

Replacing the device by its model and taking into account that networks A and B (Fig. 5) are linear, the general problem of Fig. 6(a) is transformed into that shown in Fig. 6(b), where Z_a, v_a, and Z_b, v_b represent the Thevenin equivalent generators of the circuits connected at gate and drain of the device, respectively.

According to Section II, the voltages $v_1(t)$ and $v_2(t)$, controlling all the nonlinearities of the circuit, must be chosen as unknown variables. Considering the capacitances $C_1(v_1, v_2)$ and $C_2(v_1, v_2)$ as current-generator-type nonlinear elements, the circuit of Fig. 6(b) can be rearranged in the way indicated in Fig. 6(c).

The currents $i_{C_1}(t)$ and $i_{C_2}(t)$ corresponding to the capacitances C_1 and C_2 are related to the unknown variables by

$$i_{C_1}(t) = C_1(v_1, v_2)\frac{dv_1(t)}{dt} \tag{10}$$

$$i_{C_2}(t) = C_2(v_1, v_2)\frac{d}{dt}[v_1(t) - v_2(t)] \tag{11}$$

while $i_m(t)$ is directly obtained from $i_m(v_1, v_2)$.

It is easy to prove that the harmonic components of the unknowns are given by

$$\begin{bmatrix} V_{1,n} \\ V_{2,n} \end{bmatrix} = \begin{bmatrix} -(Z_n + Z_{a,n}) & -Z_{a,n} & -Z_n & 1 & 0 \\ -Z_n & Z_{b,n} & -(Z_n + Z_{b,n}) & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{C_1,n} \\ I_{C_2,n} \\ I_{m,n} \\ V_{a,n} \\ V_{b,n} \end{bmatrix}. \tag{12}$$

where $I_{C_1,n}$, $I_{C_2,n}$, and $I_{m,n}$ are the Fourier coefficients of $i_{C_1}(t)$, $i_{C_2}(t)$, and $i_m(t)$, respectively, $Z_n = R + jn\omega_0 L$, $Z_{a,n}$, and $Z_{b,n}$ denote the values of $Z_a(f)$ and $Z_b(f)$, respectively, at nf_0, and $n = 0,1,2,\cdots,N$, if only N harmonics are considered.

Note that is is not necessary to compute $i_{C_1}(t)$ and $i_{C_2}(t)$ because their Fourier coefficients can be obtained by

$$I_{C_1,n} = \sum_{k=-N}^{N} j\omega_0 k C_{1,n-k} V_{1,k} \tag{13}$$

$$I_{C_2,n} = \sum_{k=-N}^{N} j\omega_0 k C_{2,n-k}(V_{1,k} - V_{2,k}) \tag{14}$$

where $C_{1,j}$ and $C_{2,j}$ are the Fourier coefficients of $C_1(v_1, v_2)$ and $C_2(v_1, v_2)$, respectively. Then, time-domain calculations are reduced to computing the instantaneous values of the different nonlinear characteristics determining their Fourier coefficients via DFT or FFT.

The resulting nonlinear system of $2(N+1)$ equations can be solved by the iteration technique described in Section II. Observe that the values of the unknowns corresponding to the first iteration are given by

$$(V_{1,n})_1 = V_{a,n} \qquad (V_{2,n})_1 = V_{b,n}, \tag{15}$$

for $n = 0,1,2,\cdots,N$.

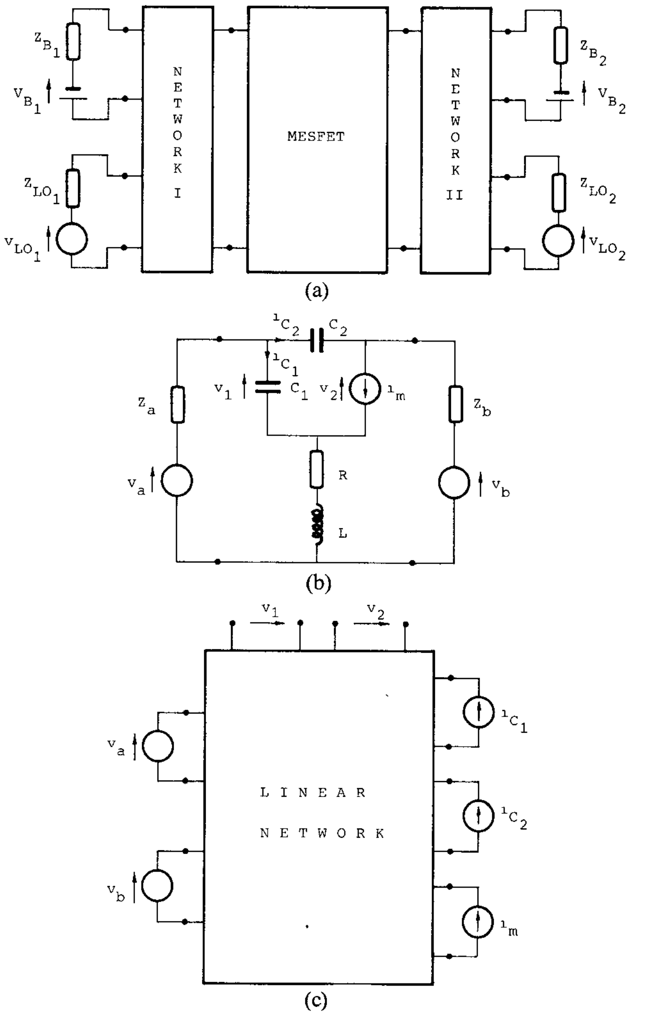

Fig. 6. (a) The general nonlinear MESFET problem (large-signal amplifiers, harmonics generators, and frequency convertors). (b) Transformed. (c) Rearranged.

C. Application: Analysis of a MESFET Frequency Doubler

To illustrate the capabilities of the proposed technique and to check its convergence characteristics, a frequency doubler at 2 GHz has been designed and analyzed.

After selecting appropriate bias conditions ($V_{GS} = -2.0$ V, $V_{DS} = 3.0$ V) near the knee of the $I_{DS} - V_{DS}$ characteristics, the analysis technique was used to determine the device response for different load conditions.

With 50 Ω at both ports, the technique has been able to handle 12 harmonics with no convergence problems [7]. For a selected incident power level the optimum load at the second harmonic was determined. The impedances presented to the device at the fundamental and harmonics other than second (six harmonics were considered) were then modified in order to study their influence on the output power at the second harmonic. It was found that they had no influence because calculated variations of this output power were less than 1 dB.

A prototype was constructed. The input network used a quarter-wave transformer to reduce the high reflection of the device at 2 GHz and presented a reactive impedance at

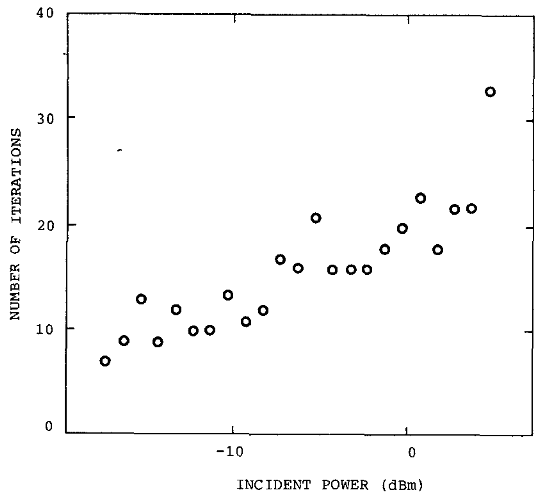

Fig. 7. Number of iterations (six harmonics) as a function of the incident power level. ($V_{GS} = -2.0$ V, $V_{DS} = 3.0$ V).

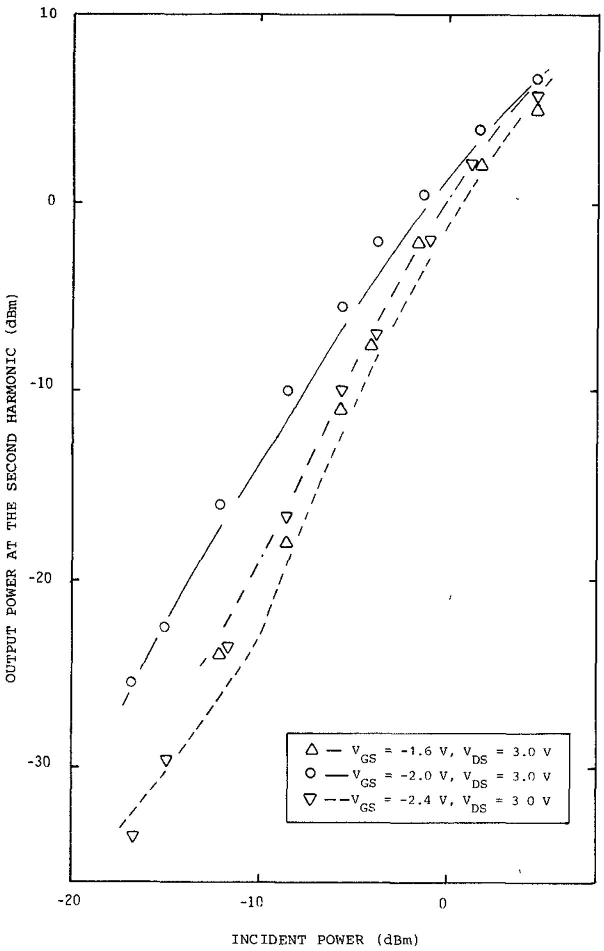

Fig. 8. Measured (points) predicted (lines) output power at the second harmonic.

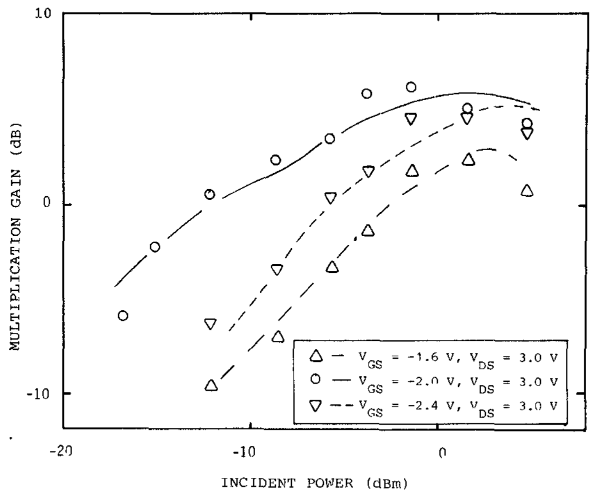

Fig. 9. Measured (points) and predicted (lines) multiplication gain.

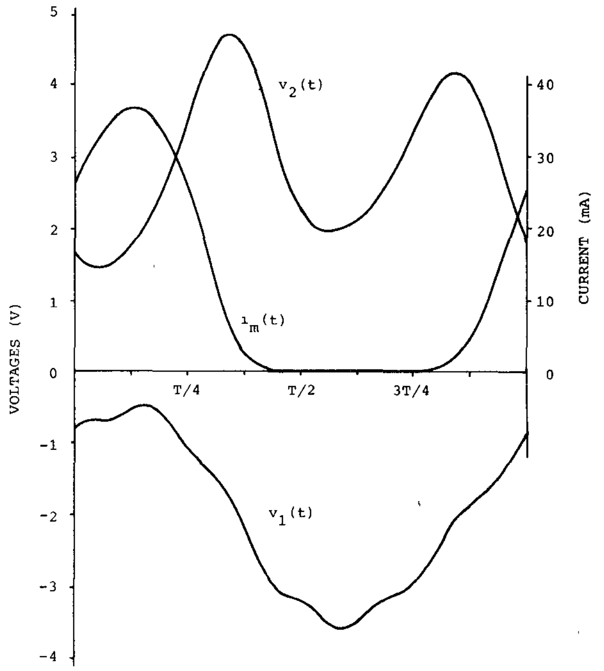

Fig. 10. MESFET frequency doubler waveforms. ($V_{GS} = -2.0$ V, $V_{DS} = 3.0$ V, $P_{inc} = 2.8$ dBm).

the second harmonic. The output network synthesized the previously determined optimum load at the second harmonic and included a stub to improve the output spectrum. The actual impedances presented by these networks to the device were measured up to the sixth harmonic and were used to compute the device response in such conditions for different incident power levels (a complete description of the measured impedances and the nonlinear MESFET model can be found in [7]).

Voltage reflection coefficients (as defined in the analysis of the pumped diode), at the terminal planes of the MESFET, were utilized as convergence parameters. Convergence was deemed to occur when they were (in magnitude) less than 0.01 (-40 dB) at every harmonic. Convergence was achieved in all the cases analyzed and the required number of iterations as a function of incident power is shown in Fig. 7. Note that no more than 35 iterations were necessary to reach the solution in all the cases. Similar performance was obtained for other bias conditions [7].

The measured and predicted output powers at the second harmonic and the multiplication gain (ratio of the second harmonic output power to the fundamental frequency input power), at three different bias conditions, are represented in Figs. 8 and 9, respectively. Similar agreement between measured and predicted characteristics has been observed up to the sixth harmonic [7]. Typical waveforms are plotted in Fig. 10.

V. CONCLUSIONS

A new method for determination of the steady-state response of nonlinear microwave circuits with periodic excitation has been described. It has been successfully applied to the analysis of a pumped diode and a MESFET frequency doubler. Its ability to handle large number of harmonics and nonlinearities has been confirmed.

The reduction in time-domain calculations obtained by systematic selection of the unknown variables, and the excellent convergence characteristics shown by the proposed algorithm, enable the described method to compete advantageously with other techniques and to be a powerful tool in the design of nonlinear microwave circuits, including GaAs monolithic integrated circuits.

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